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Predicting delayed stability of metals at constant high temperatures

Jinlai Zhou [a], Bo Peng [b], Qinghe Yao [a,b,*], Gengchao Yang [a,**]

[a] School of Aeronautics and Astronautics, Sun Yat-Sen University, Guangzhou, 510275, China
[b] School of Integrated Circuits, Sun Yat-Sen University, Shenzhen, 518107, China

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ABSTRACT

In this paper, we propose and validate a general constitutive behavior framework, which predicts the structural delayed stability of metals in constant temperature (from room temperature to high temperature) engineering environments through experimental mechanics methods. Through experiments and energy analysis, it is revealed that the yield of solid materials does not depend on stress, but only on strain. The viscous effect caused by constant high temperature promotes the transformation of the linear elastic-plastic zones of the metal into linear viscoelastic and viscoplastic zones. The infinite linear elastic-plastic constitutive behavior is the only physical boundary for measuring structural stability, and the yield strength and fracture strength (engineering allowable stress) at any constant high-temperature are defined at this constitutive boundary. Furthermore, it reveals that viscoelasticity and viscoplasticity are strengthening superposition constitutive relation rather than independent superposition, thus establishing a general stress relaxation and creep constitutive behavior. We propose Maxwell fluid and Kelvin solid visco-elasto-plastic models to characterize the complete general relaxation and creep characteristics, from linear viscoelastic zone to viscoplastic zone. The establishment of a general constitutive behavior framework is crucial for the development and engineering applications of constant temperature solid mechanics, and has profound scientific significance.

1. Introduction

When a metal is subjected to step loading and constant stresses at constant high temperatures, a continuously increasing strain is often induced, which is a process known as creep [1]. A large number of failures in materials and components working at constant high temperatures are attributed to creep or its combination with other degradation processes [2,3], and billions of dollars are spent annually to repair and replace parts in advanced equipment [4]. The increasing demand for higher fuel efficiency and reliability in nuclear reactors, turbines, and chemical industry equipment requires continuous exploration of the creep stability and failure performance of advanced alloys [3,5]. Taking the fast breeder reactor as an example, its operating temperature is about 550 °C, and its design life is 40 years or longer [6,7]. The stability and failure assessment of viscoelastic structures is crucial for the structural safety of components such as nuclear reactors at constant high temperatures.

However, determining the stability of a viscoelastic structure is a difficult task [8]. The essence is the lack of a theoretical framework for predicting the future stability of these systems. Although in most cases equations can be derived to describe the viscoelastic behavior, the mechanism of delayed instability in viscoelastic solids is still poorly understood [8]. This is also an important reason for the limited development of constant high-temperature solid mechanics of metals over the years, leading to a lack of theoretical support for the engineering allowable stress of metals at constant high temperatures.

The constitutive model describes the relation between material stress, strain, and strain rate, which can be divided into elasticity, plasticity, viscosity, and any combination of them [9–13]. There are generally two types of traditional creep constitutive modeling methods. The first type is mathematical modeling, which only starts from a mathematical perspective. Some studies developed new mathematical equations based on power-law functions to comprehensively characterize the viscoelastic and viscoplastic creep fracture behavior [14–18]. However, they overlooked the physical differences between viscoelastic creep and viscoplastic creep.

The second creep constitutive modeling method is the independent superposition of viscoelastic and viscoplastic models, where the

* Corresponding author. School of Aeronautics and Astronautics, Sun Yat-Sen University, Guangzhou, 510275, China.
** Corresponding author.
E-mail addresses: yaoqhe@mail.sysu.edu.cn (Q. Yao), yanggch8@mail.sysu.edu.cn (G. Yang).

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viscoelastic and viscoplastic models are independent of each other and do not affect each other [12,14]. However, the independent superposition modeling method of viscoelasticity and viscoplasticity makes it theoretically impossible to describe the stress relaxation behavior in the viscoplastic zone. Therefore, the independent superposition modeling method of viscoelasticity and viscoplasticity has certain limitations.

The superposition of viscoelastic and viscoplastic models also involves the definition of the critical yield criterion between these two states. Traditional mechanics of room - temperature metal materials holds that the yield of metals depends on stress. The yield stress is naturally used as the boundary between the viscoelastic and viscoplastic zones [12,14]. In classical elastoplastic theory, the yield surface [19] in three-dimensional space is often used as the boundary between elasticity and plasticity. It is one of the fundamental concepts in plastic mathematical theory [20]. However, there is still no fully recognized theory for the critical yield between viscoelasticity and viscoplasticity. Further research is needed to understand the critical yield between viscoelasticity and viscoplasticity, including experimental exploration, model assumptions, and simulation applications.

Currently, the understanding of structural stability is still limited to a qualitative perspective. Low-stress creep maintains stability, while high-stress creep leads to fracture. However, the engineering allowable stress between low-stress creep stability and high-stress creep fracture has not been determined. For example, Cao et al. [21] simulated the creep stability under low-stress (42; 64 and 97 MPa) and creep fracture behavior under high-stress (198 and 258 MPa) from the perspective of molecular dynamics. However, the creep stability between the two cannot be determined (143 MPa). It is not possible to quantitatively predict the creep mechanical response covering the entire mechanical load area.

Some scholars have attempted to solve the stability problem of viscoelastic structures from the perspective of numerical simulation, and have proposed numerous concepts of mathematical stability. These phenomena are defined as temporary bistability [22], pseudo bistability [23], creep buckling [24,25], and locally stable configuration stationarity [8]. However, from a mathematical perspective, no matter how complex the equation is, it is almost impossible to solve the physical problem of the stability of viscoelastic structures. Furthermore, the issue of structural stability is essentially a problem of determining engineering allowable stress. It is imperative to find the boundary of physical stability, and the engineering allowable stress must also be obtained through experiments. Therefore, we believe that establishing a general constitutive behavior through experiments is the key to solving the stability problem of viscoelastic structures.

In this paper, the yield strain and infinite linear elastic-plastic static stable boundary are experimentally determined. A general constitutive behavior based on classical linear elastic-plastic theory is established. According to our proposed general constitutive behavior, the yield strength and fracture strength of metals at any constant temperature are defined. In this background, the criteria for determining the stability and failure of viscoelastic structures is established. Furthermore, new linear viscoelastic strengthening relaxation and creep constitutive behaviors are defined in the viscoplastic zone, proving that viscoelasticity and viscoplasticity are not independent of each other, and pointing out a new direction for strengthening superposition constitutive modeling. Finally, the Maxwell fluid and Kelvin solid visco-elasto-plastic models are put forward to characterize the linear viscoelastic strengthening relaxation and creep constitutive behavior of metals.

2. Materials and methods

2.1. Preparation of specimens

The sample was prepared using 45 # steel material. The chemical composition of 45 # steel is shown in Table 1. The sample is using the $l = 5d$, gauge length $l = 30$ mm, and diameter $d = 6$ mm cylinder. The experimental instrument adopts an Instron Division of ITW Limited (UK) fatigue testing system (model 100KN/8801), with a temperature control range of 200 °C–1000 °C. Instrument displacement resolution is 0.01 μm. The force resolution is 0.001N, and the accuracy meets the experimental requirements (Fig. 1).

Table 1
Chemical compositions (wt.%) of the 45 # steel material.

C	Si	Mn	P	S	Cr	Ni	Cu	Fe
0.44	0.24	0.58	0.018	0.004	0.027	0.007	0.013	Bal.

2.2. Experiment

The experiment was divided into three groups. This includes 12 static tensile experiments (S1-S12), 4 stress relaxation experiments (R13-R16), and 5 creep experiments (C17-C21). We set zero constant stress creep during the experimental heating process and maintained it for 2 h. The goal is to ensure that the sample has no initial stress, maintains a uniform temperature distribution, and reduces the influence of other variables. Then start formal static, relaxation, and creep experiments. The relaxation/creep experiment involves step loading along with either constant strain/constant stress conditions. In actual experiments, step loading is replaced by an adequately large strain rate of 2.5×10^{-3}/s. The constant strains for relaxation experiments are 0.5 %, 1.0 %, 1.5 %, and 2.0 %. The constant stresses for creep experiments are 20 MPa, 40 MPa, 60 MPa, 80 MPa, and 100 MPa. The detailed information is shown in Table 2.

3. Experimental results

3.1. Boundary of constitutive mechanical behavior

At 550 °C, the experimental results showed that as the strain rate decreased, the static curve showed a gradually decreasing trend. Moreover, the end of the linear segment is basically on the yield strain constitutive behavior (Fig. 2a). From an experimental perspective, it has been demonstrated that the linear viscoelastic and viscoplastic zones are bounded by yield strain. It can be concluded that the yield of metals depends solely on strain.

The static curves under the two strain rate experiments of 1.0×10^{-7}/s and 5.0×10^{-8}/s basically coincide (Fig. 2b). It indicates that the strain rate range of $1.0 \times 10^{-7}/s \sim 0^{+}/s$ can approximate the theoretically infinite linear elastic-plastic constitutive behavior ($t = +\infty$). When the strain rate is very small and the overall static load time reaches about 10 days, the experimental curve shows severe fluctuations. The main reason is that the instrument slowly stretches to a value higher than the infinite linear elastic-plastic constitutive behavior, and then the stress relaxes to the infinite constitutive behavior. Afterwards, the instrument repeated this process, and the final experimental results presented periodic and fluctuating experimental data. Therefore, it is reasonable to take the lower envelope as the infinite linear elastic-plastic constitutive behavior (Fig. 2b).

The maximum quasi-static rate is 2.5×10^{-3}/s, we use this rate to approximate the theoretical step loading. According to the experimental results, under the condition of 25 °C, the instantaneous linear elastic-plastic and infinite linear elastic-plastic constitutive behavior basically coincide, indicating that this rate does not cause thermal and impact dynamic effects on the metal (Fig. 3a). This rate is reasonable, and the overall experimental duration of the approximate step loading is about 30 s. Therefore, the experimental results can approximate the theoretical instantaneous linear elastic-plastic constitutive behavior ($t = 0^{+}$). Under high temperature conditions, metals soften and their elastic modulus decreases. Therefore, the strain rate approximation of 2.5×10^{-3}/s for step loading can be extended to other constant high-temperature experimental conditions.

As is well known, metal alloys obey Hooke's law from low

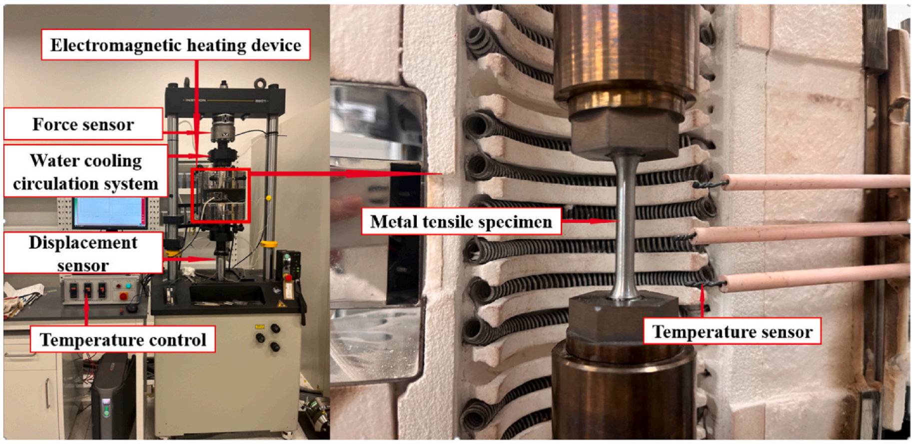

Fig. 1. High temperature uniaxial tensile test device for metal materials.

Table 2
Summary of input parameters for specimens in tensile tests.

Specimen ID	Temp.(°C)	Strain rate (1/s)	Constant stress/strain
S-1	25	2.5×10^{-3}	–
S-2	25	1.0×10^{-6}	–
S-3	400	2.5×10^{-3}	–
S-4	400	1.0×10^{-7}	–
S-5	500	2.5×10^{-3}	–
S-6	500	1.0×10^{-7}	–
S-7	550	2.5×10^{-3}	–
S-8	550	1.0×10^{-4}	–
S-9	550	1.0×10^{-5}	–
S-10	550	1.0×10^{-6}	–
S-11	550	1.0×10^{-7}	–
S-12	550	5.0×10^{-8}	–
R-13	550	–	0.5 %
R-14	550	–	1.0 %
R-15	550	–	1.5 %
R-16	550	–	2.0 %
C-17	550	–	20 MPa
C-18	550	–	40 MPa
C-19	550	–	60 MPa
C-20	550	–	80 MPa
C-21	550	–	100 MPa

temperature to room temperature. The constitutive relation of metals at different constant temperatures remains unchanged, that is, the stress and strain of metals are linear constitutive relation. The model parameters vary with constant temperature, and there are different Young's moduli at different constant temperatures. At constant high-temperature, the constitutive relation of metals remains unchanged, as fully confirmed by three sets of experiments at 400 °C (Fig. 3b)、500 °C (Figs. 3c) and 550 °C (Fig. 3d). The experimental results confirmed that constant high temperature induces the viscous effect of metals, leading to the transformation of linear elastic-plastic zones into linear visco-elastic and viscoplastic zones.

The constant room temperature linear elastic-plastic constitutive behavior is a limit of the constant high-temperature visco-elasto-plastic constitutive behavior, and the two are in a relation of opposition and unity. The experimental results confirm that the instantaneous linear elastic-plastic boundary, infinite linear elastic-plastic boundary, yield strain boundary, and fracture boundary established in this study are the constitutive behavior boundaries of metal materials. It is applicable to constant temperatures ranging from low temperature to high temperature, and has general scientific significance. The constitutive relation of metals remains unchanged at different constant temperatures. We chose a constant temperature of 550 °C to study the general stress relaxation and creep constitutive behavior.

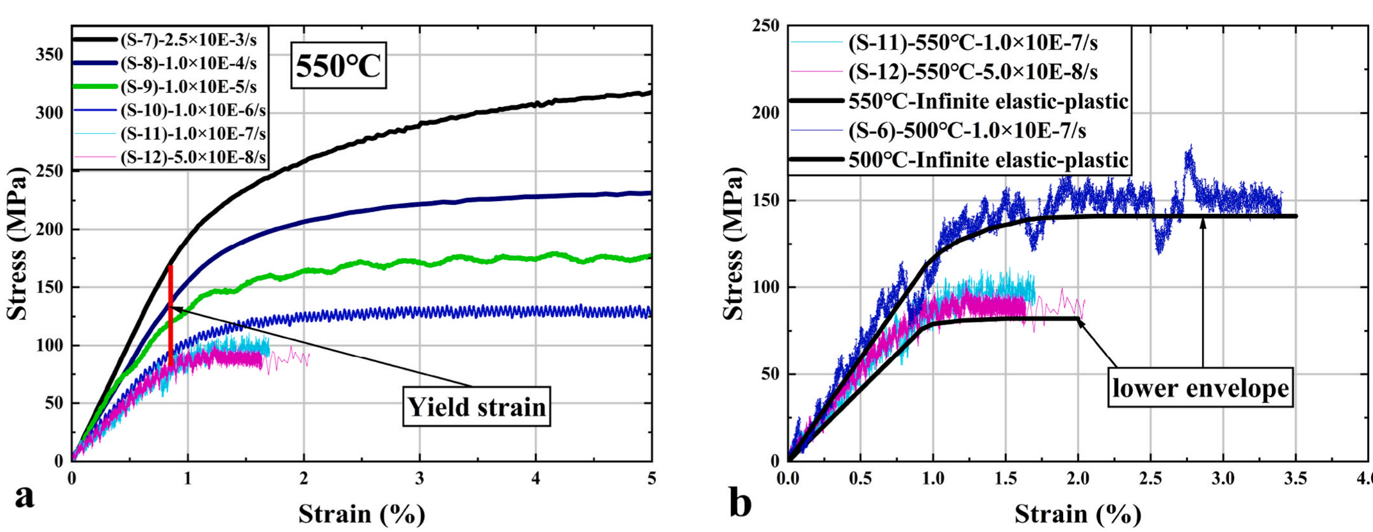

Fig. 2. Experimental verification of yield strain and source of infinite elastic-plastic boundary. (**a**). 550 °C, The end of the linear segment of the experimental curve at different rates is basically at the same yield strain. (**b**). 550 °C, the infinite linear elastic-plastic boundary is taken from the lower envelope of low strain rate (S-11; 1.0×10^{-7}/s) curve. In addition, 500 °C, the infinite linear elastic-plastic boundary is taken from the lower envelope of low strain rate (S-6; 1.0×10^{-7}/s) curve.

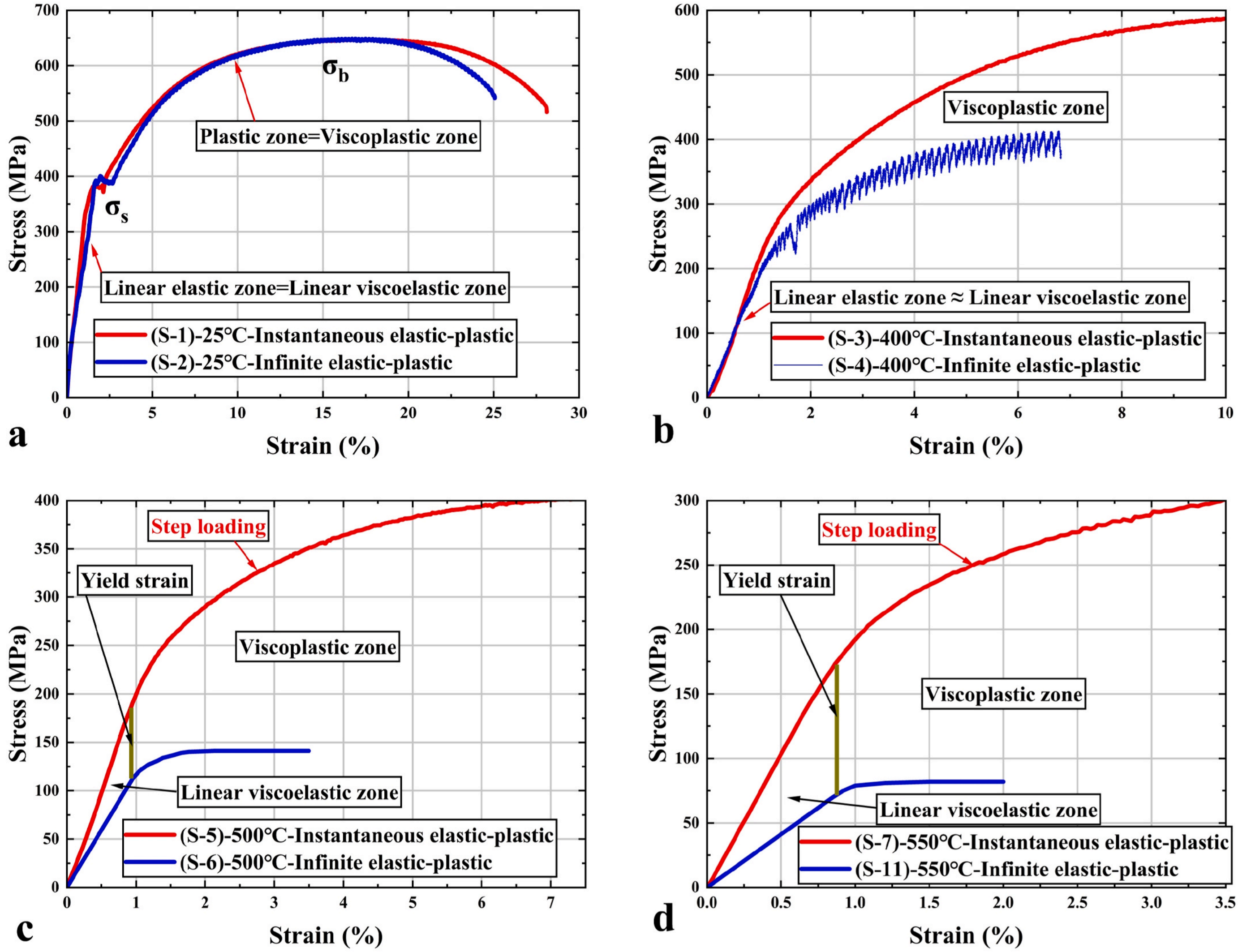

Fig. 3. The viscous effect caused by high temperature leads to the transformation of linear elastic-plastic zones into linear viscoelastic and viscoplastic zones. (**a**). 25 °C. The viscosity is 0, the linear elastic zone is equivalent to the linear viscoelastic zone, and the plastic zone is equivalent to the viscoplastic zone. (**b**). 400 °C. The plastic zone exhibits viscosity, and the plastic zone transforms into a viscoplastic zone. The linear viscoelastic zone has already appeared, but it is very close to the linear elastic zone. (**c**). 500 °C. The linear elastic zone transforms into a linear viscoelastic zone. (**d**). 550 °C. The areas of the linear viscoelastic and viscoplastic zones further expand.

3.2. Stress relaxation constitutive mechanical behavior

The stress relaxation constitutive mechanical behavior was studied at constant strain levels of 0.5 %, 1 %, 1.5 %, and 2 % at 550 °C (Fig. 4). The instantaneous linear elastic-plastic constitutive behavior occurs under step loading (Fig. 4a), and the instantaneous linear elastic-plastic constitutive behavior determines the starting point of relaxation stress. The relaxation stress exhibits exponential decay and approaches constant stress after an infinite duration (Fig. 4b). During the relaxation process, the time-dependent viscous effect gradually disappears and converges to an infinite linear elastic-plastic boundary (Fig. 4c). Essentially, step loading stores elastic strain energy and plastic deformation. Under constant strain, the stress relaxation is only phenomenological and essentially viscous consumption of elastic strain energy. From another perspective, stress relaxation constitutive behavior is the process of convergence from quasi-static unstable instantaneous linear elastic-plastic boundary to static stable infinite linear elastic-plastic boundary (Figs. 3d & 4d).

3.3. Creep constitutive mechanical behavior

The creep constitutive mechanical behavior was studied at constant stress levels of 20 MPa, 40 MPa, 60 MPa, 80 MPa, and 100 MPa at 550 °C (Fig. 5). The instantaneous elastic constitutive behavior occurs under step loading (Fig. 5a), and the instantaneous elastic constitutive behavior determines the starting point of the creep constitutive behavior. The creep strain increases exponentially and approaches a constant strain after an infinite duration (Fig. 5b). During the creep process, the time-dependent viscous effect gradually disappears and converges to an infinite linear elastic-plastic boundary (Fig. 5c). The essence of creep is the convergence from quasi-static unstable instantaneous linear elastic-plastic boundary to statically stable infinite linear elastic-plastic boundary (Figs. 3d & 5d). During the creep process, the time-dependent viscous effect gradually disappears. However, when the creep constant stress is greater than the infinite linear elastic-plastic boundary, the creep strain cannot converge to the infinite linear elastic-plastic boundary. The time-dependent viscous effect has always existed, and experimental results have demonstrated the flow characteristics of Newtonian fluids. Due to local necking of the metal, the steady-state Newtonian fluid cannot be maintained, resulting in tertiary

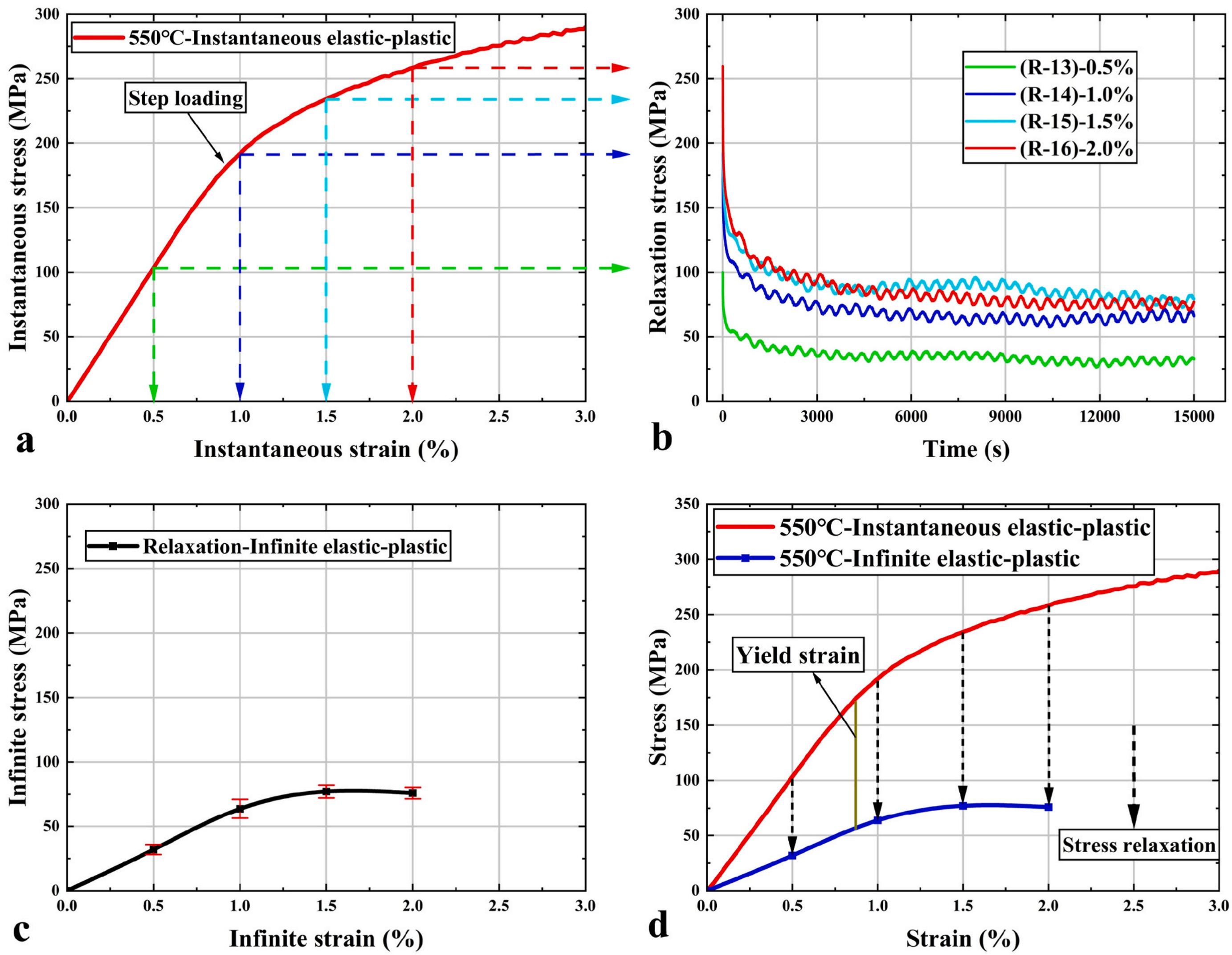

Fig. 4. General stress relaxation constitutive behavior. (**a**). Instantaneous linear elastic-plastic constitutive behavior under step loading. The instantaneous linear elastic-plastic constitutive behavior in Fig. 4a determines the starting point of the relaxation stress in Fig. 4b. (**b**). Stress relaxation occurs at constant strains of 0.5 %, 1 %, 1.5 %, and 2 %. The stress of the relaxation stress curve in Fig. 4b converges to the infinite linear elastic-plastic constitutive behavior in Fig. 4c. (**c**). The infinite linear elastic-plastic constitutive behavior. The error bar is taken from the fluctuation difference of the data in Fig. 4b. (**d**). The instantaneous linear elastic-plastic constitutive behavior undergoes stress relaxation and converges to infinite linear elastic-plastic constitutive behavior.

creep and eventually tending towards creep fracture (Fig. 5e). It has been proven from the perspective of creep that the constitutive behavior of infinite linear elastic-plastic boundaries is the key to determining creep stability and fracture failure, and this physical stability is unique.

3.4. Engineering allowable stress at different constant temperatures

The yield strength of 45 # steel at 25 °C is 388 MPa, and the fracture strength is 647 MPa (Fig. 3a). The yield strength at 400 °C is 230 MPa, and the fracture strength is 378 MPa (Fig. 3b). The yield strength at 500 °C is 120 MPa, and the fracture strength is 141 MPa (Fig. 3c). The yield strength at 550 °C is 70 MPa, and the fracture strength is 82 MPa (Fig. 3d). Therefore, the yield strength and fracture strength at any constant temperature (from low temperature to high temperature) can be defined at a static stable infinite linear elastic-plastic boundary (Fig. 5f). A set of constant room temperature experiments and three sets of constant high temperature experiments fully validated this point. The definition of engineering allowable stress at constant temperature solves the engineering problems of structural stability and fracture failure. We will refine and upgrade it into a general theoretical system in the next section.

4. Constitutive behavior and models

4.1. General constitutive behavior framework

We first define a series of physical concepts of constitutive behavior, which are the cornerstone of theoretical innovation in this study (Fig. 6).

Linear elastic: Stress and strain have the linear constitutive relation. Time-independent instantaneous elasticity and recovery behavior, characterized at the microscopic level by the stretching and recovery of atomic bonds.

Plastic: Stress and strain have the nonlinear constitutive relation. Time-independent plasticity and irreversible behavior, characterized by atomic dislocations at the microscopic level.

Linear elastic strengthening: When the load is greater than the yield strain, plastic deformation occurs, and the linear elastic zone is strengthened ($O_1A_1 \rightarrow O_2A_2$). At the microscopic level, plastic deformation of atomic dislocations occurs while the formation of new atomic bonds is stretched. Materials store more elastic strain energy (Fig. 6a).

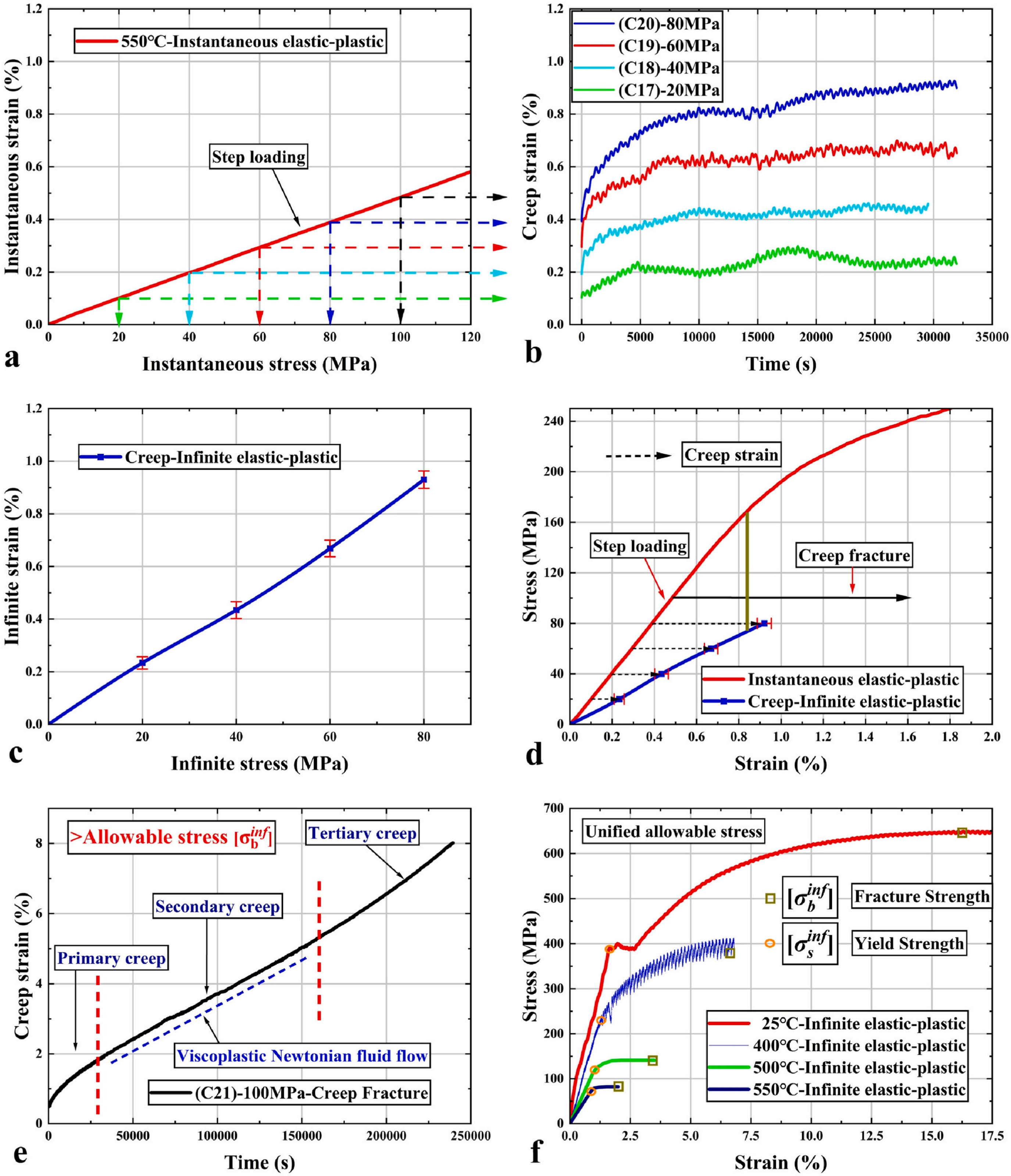

Fig. 5. General creep constitutive behavior. (**a**). Instantaneous linear elastic-plastic constitutive behavior under step loading. The instantaneous linear elastic-plastic in Fig. 5a determines the starting point of the creep strain in Fig. 5b. (**b**). Creep strain occurs at constant stress of 20 MPa, 40 MPa, 60 MPa, and 80 MPa. The strain of the creep strain in Fig. 5b converges to the infinite linear elastic-plastic constitutive behavior in Fig. 5c. (**c**). The creep strain converges to an infinite linear elastic-plastic constitutive behavior. The error bar is taken from the fluctuation difference of the data in Fig. 5b. (**d**). The instantaneous linear elastic-plastic constitutive behavior undergoes creep strain and converges to infinite linear elastic-plastic constitutive behavior. (**e**). The instantaneous linear elastic-plastic constitutive behavior cannot converge to the infinite linear elastic-plastic constitutive behavior, and the creep fracture constitutive behavior occurs. (**f**). The yield strength and fracture strength of metal materials under different temperature conditions are defined on the infinite linear elastic-plastic constitutive behavior.

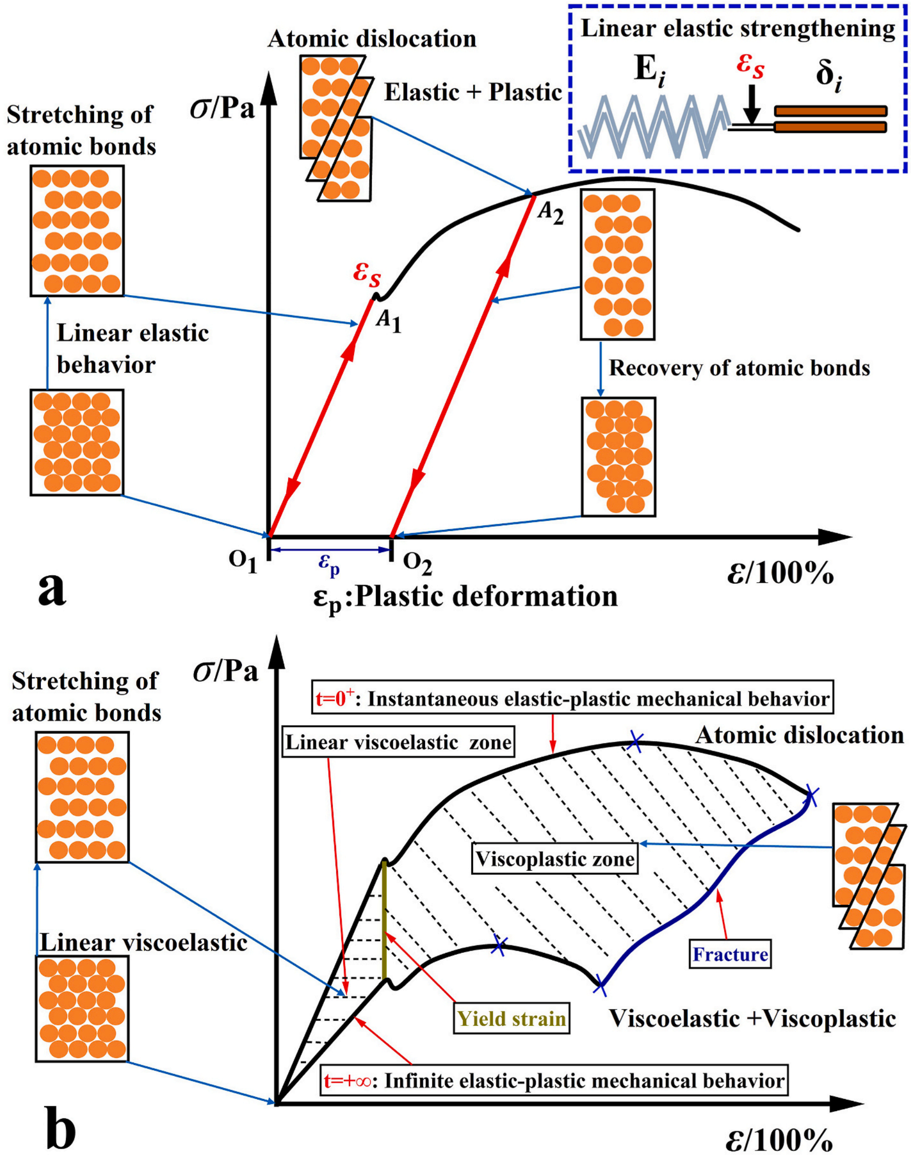

Fig. 6. General constitutive behavior of metal materials. (**a**). The classical linear elastic-plastic mechanics constitutive behavior of metal materials. This theory applies to metal alloys at a constant temperature from low temperature to room temperature. (**b**). **The contribution of this work.** We propose the boundary of high-temperature constitutive behavior for metal alloys. When the viscosity is 0, the instantaneous and infinite linear elastic-plastic constitutive behaviors overlap to form the classical linear elastic-plastic constitutive relation. Linear elastic-plastic zones are a limit in the linear viscoelastic and viscoplastic zones. This theory applies to metal alloys at a constant temperature from room temperature to high temperature.

Viscoelastic: Time-dependent hysteresis elasticity and hysteresis recovery behavior, characterized at the microscopic level by the stretching and recovery of atomic bonds.

Viscoplastic: Time-dependent viscoplasticity and irreversible behavior, characterized by atomic dislocations at the microscopic level (Fig. 6b).

Static stability: statics, no acceleration, stress and strain do not change over time.

Quasi-static instability: Quasi-static, non-dynamic, with small accelerations that can be ignored, and slow changes in stress and strain over time.

Instantaneous linear elastic-plastic ($t = 0^+$): The mechanical response of a material under a load rate of $(+\infty/s)$.

Infinite linear elastic-plastic ($t = +\infty$): The mechanical response of a material under a loading rate of $(0^+/s)$ [26].

In 1821, Cauchy proposed the concepts of stress and strain [27] and obtained the classical linear elastic-plastic constitutive behavior of metal alloys (Fig. 6a). In the following two centuries, this theory was validated through countless experiments. This constitutive behavior applies to metal alloys at constant temperatures (non viscous) from low temperature to room temperature. The phenomenological model is characterized by a series connection of springs (E_i) and friction component (δ_i) (Fig. 6a). The linear elastic-plastic constitutive behavior of metals is statically stable.

As the constant temperature increases, the viscous effect of the metal is induced, leading to a transition from linear-elastic plastic zones to linear viscoelastic and viscoplastic zones (Fig. 6b). The step loading [28] can be regarded as a static load in infinitesimal time. In an instantaneous state, "time" does not exist as a mathematical variable. From a physical perspective, the behavior of viscous mechanics requires time to characterize. Metal only exhibits instantaneous linear elastic-plastic constitutive behavior under step loading ($t = 0^+$). Instantaneous linear-elastic plastic constitutive behavior is quasi-static instability, which is one of the boundaries of constitutive behavior.

After step loading, time-dependent viscous effects are generated, resulting in quasi-static unstable viscoelastic and viscoplastic behaviors. It converges to infinite linear elastic-plastic behavior within an infinite time (t = + ∞). The viscous mechanical response is characterized by infinite time, and the time-dependent viscous effect disappears. This new constitutive behavior is defined as "infinite linear elastic-plastic constitutive behavior" [26]. It is the only static stable in constitutive behavior and also one of the boundaries of constitutive behavior. The instantaneous linear elastic behavior, infinite linear elastic behavior, and yield strain behavior are the boundaries of the linear viscoelastic zone. The instantaneous plastic behavior, infinite plastic behavior, yield strain behavior and fracture behavior are the boundaries of the viscoplastic zone.

Classical linear elastic-plastic constitutive theory is applicable to engineering applications of metallic materials at constant temperatures (low temperature to room temperature) (Fig. 6a). Classical linear elastic-plastic constitutive behavior theory does not have viscous effects. We have incorporated viscous effects into it and proposed a general constitutive behavior. Within the general constitutive behavior, the linear elastic-plastic constitutive behavior at constant room temperature (Fig. 6a) is a limit of the general constitutive behavior (Fig. 6b). The general constitutive behavior is applicable to constant temperatures ranging from low temperature to high temperature. The constitutive relation remains unchanged as the constant temperature changes from low temperature to high temperature. Only the parameters of the model vary with constant temperature. This general constitutive behavior has been fully validated by three sets of constant high temperature experiments (Fig. 4).

4.2. Linear viscoelastic stress relaxation and creep

In 1867, Maxwell discovered the linear viscoelastic stress relaxation, characterized by a linear constitutive relation between relaxation stress and constant strain. In 1875, Kelvin discovered the linear viscoelastic solid creep, characterized by a linear constitutive relation between creep strain and constant stress. In the general constitutive behavior, the linear viscoelastic zone is a similar triangle of linear relations, perfectly unifying the linear viscoelastic stress relaxation and creep constitutive behavior (Fig. 7a) [26].

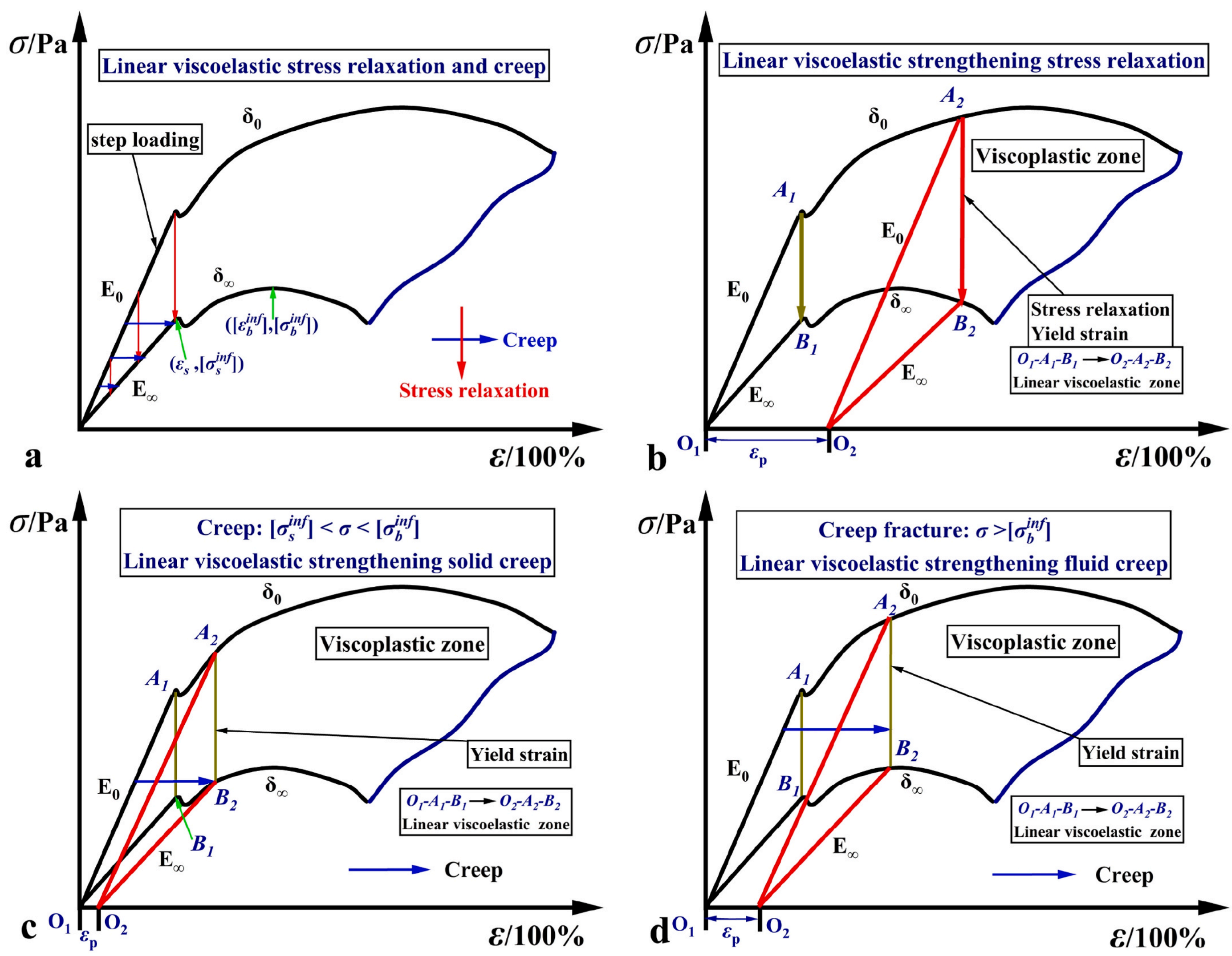

Fig. 7. The general stress relaxation and creep constitutive behavior of metal materials. (**a**). The yield point (ε_s,[σ_s^{inf}]) and fracture point ([ε_b^{inf}],[σ_b^{inf}]) are defined on an infinite linear elastic-plastic constitutive behavior. Linear viscoelastic stress relaxation and creep constitutive behavior in linear viscoelastic zone. The linear viscoelastic zone is strengthened ($O_1A_1B_1 \rightarrow O_2A_2B_2$). (**b**). Linear viscoelastic strengthening stress relaxation constitutive behavior in viscoplastic zone. The linear viscoelastic zone is strengthened ($O_1A_1B_1 \rightarrow O_2A_2B_2$). (**c**). Linear viscoelastic strengthening solid creep constitutive behavior in viscoplastic zone. The linear viscoelastic zone is strengthened ($O_1A_1B_1 \rightarrow O_2A_2B_2$). (**d**). Linear viscoelastic strengthening fluid creep constitutive behavior in viscoplastic zone. The material loses stability and subsequently undergoes creep fracture behavior. The linear viscoelastic zone is strengthened ($O_1A_1B_1 \rightarrow O_2A_2B_2$).

4.3. Linear viscoelastic strengthening stress relaxation

The essence of stress relaxation is the consumption of elastic strain energy by viscosity. Under step loading, instantaneous linear elastic-plastic constitutive behavior occurs (Fig. 7b). The plastic deformation of atomic dislocations only occurs under step loading. Under constant strain, external forces no longer do work on the material, and atomic dislocations no longer occur. Stress relaxes to infinite linear elastic-plastic constitutive behavior over time (Fig. 7a and b). Therefore, yield depends only on strain (Appendix A). We found that the "yield strain(ε_s)" is the boundary between the stretching of atomic bonds in the linear viscoelastic zone and the atomic dislocations in the viscoplastic zone (Fig. 7b). At the same time, the viscoplasticity strengthens linear viscoelasticity (Fig. 7b), and there is no physical concept of "viscoplastic stress relaxation". The discovered constitutive behavior "linear viscoelastic strengthening stress relaxation" is defined.

- When the constant strain is less than the yield strain (ε_s). The plastic deformation caused by linear viscoelastic stress relaxation is $\varepsilon_p = 0$. The linear viscoelastic zone is not strengthening ($O_1A_1B_1 \rightarrow O_1A_1B_1$) (Fig. 7a).
- When the constant strain is greater than the yield strain (ε_s). The plastic deformation caused by linear viscoelastic strengthening stress relaxation is $\varepsilon_p > 0$. The linear viscoelastic zone is strengthened ($O_1A_1B_1 \rightarrow O_2A_2B_2$) (Fig. 7b).

Our contribution is to determine that the stress relaxation in the viscoplastic zone is a "linear viscoelastic strengthening stress relaxation" constitutive behavior (Fig. 7b). Based on Maxwell's discovery of linear viscoelastic stress relaxation in the linear viscoelastic zone (Fig. 7a), the general constitutive behavior of stress relaxation has been established.

Obviously, the plastic deformation caused by linear elastic behavior is $\varepsilon_p = 0$. The plastic deformation caused by linear elastic strengthening constitutive behavior is $\varepsilon_p > 0$. Linear elastic constitutive behavior is the limit of linear elastic strengthening constitutive behavior (Fig. 6a). The two are in a relation of opposition and unity. Linear viscoelastic stress relaxation is the limit of linear viscoelastic strengthening stress relaxation. The two are in a relation of opposition and unity. The stress relaxation at constant room constant temperature is 0. When the constant temperature drops from high to constant room constant temperature. As viscosity disappears, linear viscoelastic stress relaxation (Fig. 7a) transforms into linear elastic constitutive behavior (Fig. 6a). Linear viscoelastic strengthening stress relaxation (Fig. 7b) transforms into linear elastic strengthening constitutive behavior (Fig. 6a). The essence of stress relaxation constitutive behavior is the process of convergence from quasi-static unstable instantaneous linear elastic-plastic boundary to static stable infinite linear elastic-plastic boundary.

4.4. Engineering allowable stress for structural stability

In the general constitutive behavior framework (Fig. 6b), the engineering allowable stress of metals at any constant temperature is defined as the static stable infinite linear elastic-plastic boundary (Fig. 7a), which is the yield strength $[\sigma_s^{inf}]$ and fracture strength $[\sigma_b^{inf}]$. This general strength theory definition applies to constant temperatures from low temperature to high temperature (Fig. 5f).

4.5. Linear viscoelastic strengthening solid and fluid creep

The yield strength $[\sigma_s^{inf}]$ and fracture strength $[\sigma_b^{inf}]$ are defined at the static stable infinite linear elastic-plastic boundary. We classify creep into three levels.

- Level "I" creep (Fig. 7a): Creep constant stress is at $0 \sim [\sigma_s^{inf}]$. It is called linear viscoelastic solid creep. The creep strain converges to the static stable infinite linear elastic boundary. The linear viscoelastic zone is strengthened to 0 ($O_1A_1B_1 \rightarrow O_1A_1B_1$).
- Level "II" creep (Fig. 7c): Creep constant stress is at $[\sigma_s^{inf}] \sim [\sigma_b^{inf}]$. When the creep strain is less than the yield strain (ε_s), it is called linear viscoelastic creep. When the creep strain is greater than the yield strain (ε_s), it is called viscoplastic solid creep, also known as linear viscoelastic strengthening solid creep ($O_1A_1B_1 \rightarrow O_2A_2B_2$). The creep strain converges to the static stable infinite plastic boundary.
- Level "III" creep (Fig. 7d): Creep constant stress is greater than $[\sigma_b^{inf}]$. When the creep strain is less than the yield strain (ε_s), it is called linear viscoelastic creep. When the creep strain is greater than the yield strain (ε_s), it is called viscoplastic fluid creep, also known as linear viscoelastic strengthening fluid creep ($O_1A_1B_1 \rightarrow O_2A_2B_2$). The creep strain does not converge, resulting in a physical phenomenon of creep fracture.

Our contribution is to determine that the creep in the viscoplastic zone is a constitutive behavior of "linear viscoelastic strengthening creep" (Fig. 7c and d). Based on Kelvin's discovery of linear viscoelastic creep in the linear viscoelastic zone (Fig. 7a), the general creep constitutive behavior has been established.

Compared to the non-existent "viscoplastic stress relaxation" constitutive behavior, it is evident that there exists a physical concept of "viscoplastic creep" [13] constitutive behavior. The essence of creep constitutive behavior is the process of convergence from quasi-static unstable instantaneous linear elastic-plastic boundary to static stable infinite linear elastic-plastic boundary. The reason for creep fracture is that the creep strain cannot converge to the static stable infinite linear elastic-plastic boundary, and the viscous effect triggers the flow behavior of Newtonian fluids. This quasi-static instability persists until the material fractures. We have discovered and created a criterion for determining the creep stability and failure of all viscoelastic solid materials.

The creep at constant room temperature is 0 (Fig. 6a). When the constant temperature drops from a high temperature to room temperature, as viscosity disappears, linear viscoelastic creep (Fig. 7a) transforms into linear elastic constitutive behavior (Fig. 6a). Linear viscoelastic strengthening creep (Fig. 7c and d) transforms into linear elastic strengthening constitutive behavior (Fig. 6a). The linear elastic behavior is a limit of linear viscoelastic creep, and the two are in a relation of opposition and unity.

4.6. Relaxation and creep constitutive models

Maxwell proposed the Maxwell viscoelastic fluid model, which consists of a series of dampers (η_i^r) and springs (E_i^r) [27,29–31]. We propose a new Maxwell visco-elasto-plastic fluid model by combining Maxwell's linear viscoelastic theory and classical linear elastic-plastic theory. Which consists of dampers (η_i^r), springs (E_i^r), and friction components (δ_i^r) in series (Fig. 8a).

- When the constant strain is less than the yield strain (ε_s), the Maxwell visco-elasto-plastic model is equivalent to the Maxwell viscoelastic fluid model. Under step loading, dampers and friction components do not work, and only the spring undergoes instantaneous linear elastic behavior. Under constant strain, the damper completely consumes the elastic strain energy of the spring, characterizing linear viscoelastic stress relaxation (Fig. 7a).
- When the constant strain is greater than the yield strain (ε_s). Under step loading, the damper does not work, and the spring and friction components undergo instantaneous linear elastic-plastic constitutive behavior. At this point, the friction component strengthens the spring, characterizing the linear elastic strengthening constitutive behavior (Fig. 6a). Under constant strain, the friction component does not work, and the damper completely consumes the elastic

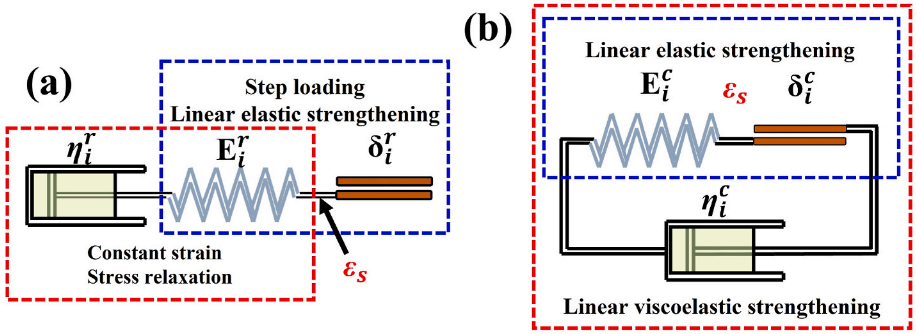

Fig. 8. General constitutive model theory. (**a**). The Maxwell visco-elasto-plastic fluid relaxation constitutive model. (**b**). The Kelvin visco-elasto-plastic solid creep constitutive model.

strain energy of the spring, characterizing linear viscoelastic strengthening stress relaxation (Fig. 7b).

Kelvin proposed the Kelvin viscoelastic solid model (also known as the Voigt model), which consists of springs (E_i^c) and dampers (η_i^c) in parallel [27,32]. We propose a new Kelvin visco-elasto-plastic solid model by combining Kelvin's linear viscoelastic theory with classical linear elastic-plastic theory. Which consists of springs (E_i^c) and friction components (δ_i^c) connected in series and with parallel dampers (η_i^c) (Fig. 8b).

- When the creep strain is less than the yield strain (ε_s) under constant stress, the Kelvin visco-elasto-plastic model equivalent to the Kelvin viscoelastic solid model, characterizing linear viscoelastic solid creep.
- When the creep strain is greater than the yield strain, the friction part strengthens the spring (Fig. 6a). The Kelvin visco-elasto-plastic model characterizes the constitutive behavior of linear viscoelastic creep to linear viscoelastic strengthening solid creep (viscoplastic solid creep).

The generalized Maxwell visco-elasto-plastic model (Fig. 9) consists of an infinite linear elastic-plastic model and n Maxwell visco-elasto-plastic fluid models in parallel. The stress of Maxwell visco-elasto-plastic fluid model decays to 0 over an infinite period of time. The stress of the overall model converges to a statically stable infinite linear elastic-plastic model $(E_\infty + \delta_\infty)$. The generalized Kelvin visco-elasto-plastic model (Fig. 10) consists of an instantaneous linear elastic-plastic model, n Kelvin visco-elasto-plastic solid models, and a visco-plastic Newtonian fluid model in series.

4.7. Constitutive equations

The instantaneous linear elastic-plastic constitutive behavior is described by eq (1):

$$\begin{cases} \sigma = E_0 \bullet \varepsilon, (0 < \varepsilon \le \varepsilon_s) \\ \sigma = K_0 - l_0 \bullet m_0^\varepsilon, (\varepsilon > \varepsilon_s) \end{cases} \tag{1}$$

Among them, E_0 is the instantaneous elastic modulus, K_0, l_0, and m_0 are the instantaneous plastic parameters, ε_s is the yield strain.

The infinite linear elastic-plastic constitutive behavior is described by eq (2):

$$\begin{cases} \sigma = E_\infty \bullet \varepsilon, (0 < \varepsilon \le \varepsilon_s) \\ \sigma = K_\infty - l_\infty \bullet {m_\infty}^\varepsilon, (\varepsilon > \varepsilon_s) \end{cases} \tag{2}$$

Among them, E_∞ is the infinite elastic modulus, K_∞, l_∞, and m_∞ are the infinite plastic parameters, ε_s is the yield strain. The continuity between linear elasticity and plasticity is explained in [Appendix B eq (B4)].

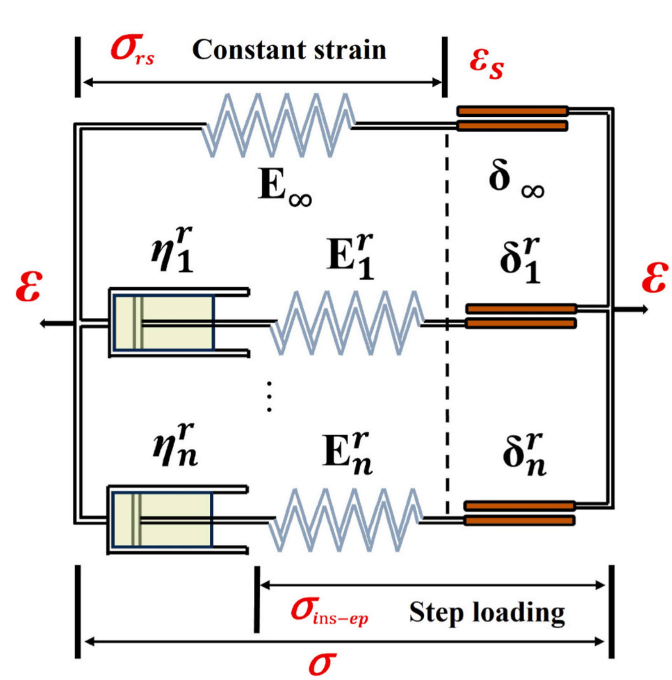

Fig. 9. The generalized Maxwell visco-elasto-plastic constitutive model. The constant strain is less than the yield strain, and the friction part does not work, indicating linear viscoelastic stress relaxation. Constant strain greater than yield strain characterizes linear viscoelastic strengthening stress relaxation. $\boldsymbol{\sigma}$: Total stress, $\boldsymbol{\varepsilon}$: Total strain, $\boldsymbol{\sigma_{ins-ep}}$: Instantaneous elastic plastic stress, $\boldsymbol{\sigma_{rs}}$: Relaxation stress. $\boldsymbol{\varepsilon_s}$: Yield strain.

The relaxation modulus of the generalized Maxwell visco-elasto-plastic model is described by eq (3):

$$G(t) = E_\infty + \sum_{i=1}^{n} E_i e^{-\frac{t}{\tau_i}} \tag{3}$$

Among them, E_∞ is the infinite elastic modulus, E_i and η_i is the elastic modulus and viscosity coefficient in the i-th Maxwell visco-elasto-plastic model, $\tau_i = \frac{\eta_i}{E_i}$ is the relaxation time. Relaxation time is a key parameter for measuring the conversion of relaxation stress to constant stress in material (45# steel).

The creep strain of the generalized Kelvin visco-elasto-plastic constitutive model is described by eq (4):

$$\varepsilon(t) = \varepsilon_{ves}(t) + \varepsilon_{vps}(t) + \varepsilon_{vpf}(t) \tag{4}$$

Among them, $\varepsilon_{ves}(t)$、 $\varepsilon_{vps}(t)$、 $\varepsilon_{vpf}(t)$ are the creep strain of visco-elastic solid, viscoplastic solid, and viscoplastic fluid, as shown in eqs.

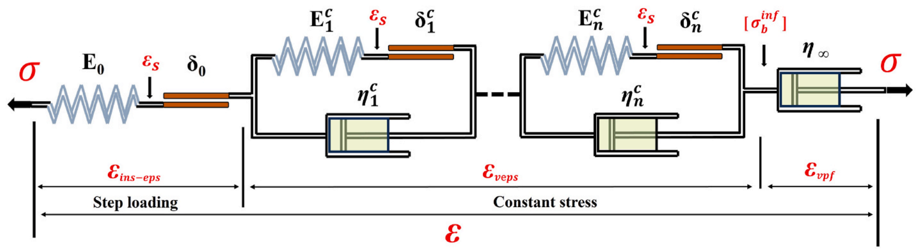

Fig. 10. The generalized Kelvin visco-elasto-plastic constitutive model. σ: Total stress, ε: Total strain, $\varepsilon_{ins-eps}$: Instantaneous elastic plastic solid strain, ε_{veps}: Visco-elasto-plastic solid strain, ε_{vpf}: Viscoplastic fluid strain. ε_s: Yield strain.

(5)–(7) (Appendix B):

$$\varepsilon_{ves}(t) = \frac{\sigma}{E_0} + \sum_{i=1}^{n} \frac{\sigma}{E_i}\left(1 - e^{-\frac{t}{\lambda_i}}\right) \quad (5)$$

$$\varepsilon_{vps}(t) = \sum_{i=1}^{n} \frac{1}{\ln m_i} \times \left[\ln \frac{K_i - \sigma}{l_i} - \ln\left(1 - m_i^{\frac{K_i-\sigma}{\eta_i} \times t}\right)\right] \quad (6)$$

$$\varepsilon_{vpf}(t) = \frac{\sigma}{\eta_\infty} \times \mathrm{t} \quad (7)$$

Among them, E_0 is the instantaneous elastic modulus, E_i and η_i are the elastic modulus and viscosity coefficient in the i-th Kelvin visco-elasto-plastic model, $\lambda_i = \frac{\eta_i}{E_i}$ is the creep retardation time. Creep retardation time is a key parameter for measuring the conversion of creep strain to constant strain in material (45# steel).

The linear viscoelastic solid creep constitutive behavior is characterized by eq (5). K_i, l_i, and m_i are the plastic parameters in the i-th Kelvin visco-elasto-plastic solid model. The (linear viscoelastic solid creep (ε_s)→ linear viscoelastic strengthening solid creep) constitutive behavior is characterized by the superposition of eq (5) and eq (6). η_∞ is the viscosity coefficient of the viscoplastic Newtonian fluid model. The (linear viscoelastic solid creep (ε_s)→ linear viscoelastic strengthening fluid creep) constitutive behavior is characterized by the superposition of eqs. (5)–(7). The viscoplastic solid and viscoplastic fluid models only play a strengthening superimposed role when they are greater than the yield strain (ε_s).

5. Comparison of experimental results and model predictions

5.1. Relaxation constitutive behavior and model prediction

Fig. 11a shows the strengthening of the linear viscoelastic zone ($O_1A_1B_1 \rightarrow O_2A_2B_2 \rightarrow O_3A_3B_3 \rightarrow O_4A_4B_4$) (Fig. 7b). 0.5 % belong to the linear viscoelastic stress relaxation ($\varepsilon_s = 0.85\%$). 1 %, 1.5 %, and 2 % belongs to the linear viscoelastic strengthening stress relaxation. Constant strains of 0.5 %, 1 %, 1.5 %, and 2 % resulted in plastic deformation of approximately 0 %, 0.04 %, 0.33 %, and 0.73 %, respectively. The relaxation moduli corresponding to constant strains of 0.5 %, 1 %, 1.5 %, and 2 % are equivalent to constant strains of 0.5 %, 0.96 %, 1.17 %, and 1.27 %. Its linear viscoelastic relaxation modulus is in good agreement with the model (eq. (3)) (Fig. 11c). Fig. 11b shows the instantaneous and the infinite linear elastic-plastic constitutive behavior, which are in good agreement with the model (eqs. (1) and (2)). The parameters of the generalized Maxwell visco-elasto-plastic model are shown in Table 3. The linear viscoelastic strengthening constitutive behavior follows a specific geometric law. The application of geometric laws can quickly find static stable infinite linear elastic-plastic boundary and predict the engineering allowable stress ($[\sigma_s^{inf}]$ & $[\sigma_b^{inf}]$) of metals at any constant temperature.

5.2. Creep constitutive behavior and model prediction

According to the experimental results, the yield strength $[\sigma_s^{inf}] = 70$ MPa and fracture strength $[\sigma_b^{inf}]$ = 82 MPa of 45 # steel at 550 °C (Fig. 3d). 20 MPa, 40 MPa, and 60 MPa belong to the linear viscoelastic solid creep (creep strain $< \varepsilon_s = 0.85\%$). 80 MPa belongs to the (linear viscoelastic creep (ε_s)→ linear viscoelastic strengthening solid creep), while 100 MPa belongs to the (linear viscoelastic creep (ε_s)→ linear viscoelastic strengthening fluid creep).

Fig. 12a shows a good agreement between the linear viscoelastic solid creep constitutive behavior and the generalized Kelvin viscoelastic solid model (eq. (5)). The parameters of the linear viscoelastic solid model have been determined. Based on the determined parameters of the linear viscoelastic solid model, the parameters of the viscoplastic solid model were obtained by fitting under the condition that the creep strain is greater than the yield strain (ε_s). Fig. 12b shows a good agreement between the (linear viscoelastic creep → linear viscoelastic strengthening solid creep) constitutive behavior and the generalized Kelvin visco-elasto-plastic solid model (eqs. (5) and (6)). Based on the determined parameters of the linear viscoelastic solid model and the viscoplastic solid model, the parameters of the viscoplastic fluid model were fitted under the condition that the creep strain was greater than the yield strain (ε_s). Fig. 12c shows a good agreement between the (linear viscoelastic creep → linear viscoelastic strengthening fluid creep) constitutive behavior and the generalized Kelvin visco-elasto-plastic fluid model (eqs. (5)–(7)). The parameters of the generalized Kelvin visco-elasto-plastic model are shown in Table 4.

6. Discussion

Determining the stability of a viscoelastic structure is a difficult task. Seemingly stable conformations of viscoelastic structures may gradually creep until their stability is lost, while a discernible creeping in viscoelastic solids does not necessarily lead to instability [8]. Currently, The understanding of structural stability only stays at the qualitative analysis level of maintaining stability under low-stress creep and tending towards fracture under high-stress creep instability. For example, Cao et al. [21] conducted molecular dynamics simulations on the creep characteristics of metallic glass under constant temperature (0.68 Tg) conditions. Among them, constant low-stress (42; 64; 97 MPa) maintains creep stability, while constant high-stress (198; 258 MPa) leads to creep instability and creep fracture. However, the creep stability of metallic glass is uncertain when the creep stress is around 143 MPa. The essence of the long-standing problem that has plagued people is the lack of a theoretical framework to predict the future stability of these systems [8].

The key to our analysis of stability in viscoelastic materials is the discovery of a new stationarity property. Previous researchers have discovered a series of mathematical stability phenomena from the perspective of numerical simulation [8,22–25]. Unlike many mathematical stability concepts, this study determines from an experimental perspective that physical stability is unique. Any static constitutive behavior (including relaxation and creep) either converges to an infinite

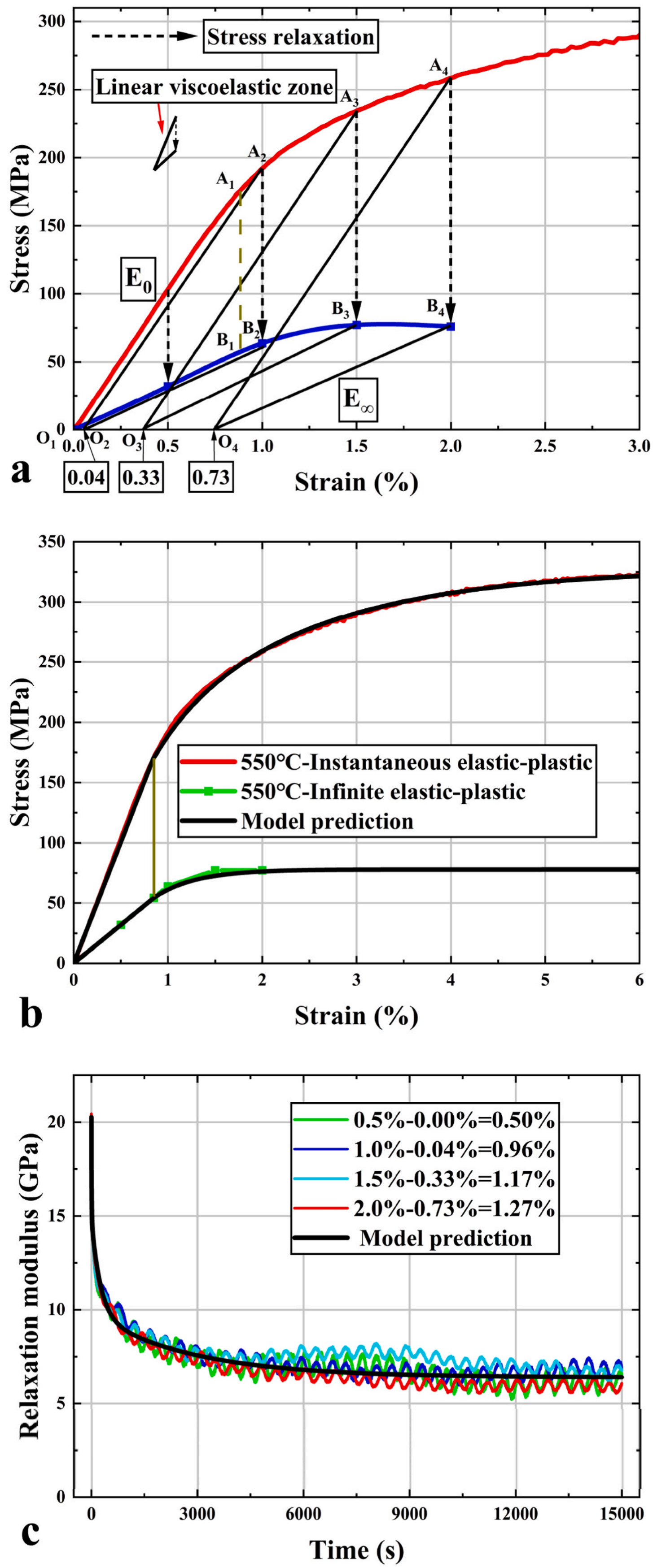

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Fig. 11. Relaxation constitutive behavior and model prediction. (**a**). The experiment verified the linear viscoelastic strengthening stress relaxation constitutive behavior. The strengthening of the linear viscoelastic zone ($O_1A_1B_1 \rightarrow O_2A_2B_2 \rightarrow O_3A_3B_3 \rightarrow O_4A_4B_4$) is an approximate result of theoretical prediction. (**b**). The comparison between the prediction by instantaneous and infinite linear elastic-plastic models and the experimental data. (**c**). The comparison between the prediction by the generalized Maxwell visco-elasto-plastic model and the experimental data in terms of the time evolution of relaxation modulus.

Table 3
Generalized Maxwell visco-elasto-plastic model parameters ($\varepsilon_s = 0.85\%$).

δ_∞			E_∞ (MPa)	δ_1^r		
K_∞ (MPa)	l_∞ (MPa)	m_∞ (100 %)		K_1 (MPa)	l_1 (MPa)	m_1 (100 %)
93.5942	167.867	5.52018E-100	6389.61	236.725	222.054	8.46717E-27
E_1^r (MPa)	η_1^r (MPa •s)	E_2^r (MPa)	η_2^r (MPa •s)	E_3^r (MPa)	η_3^r (MPa •s)	
5099.85	1,279,226	5370.36	38399056	3399.21	9648492	

linear elastic-plastic static stable boundary or does not converge, but tends to fracture. Stress relaxation (Fig. 4) and creep (Fig. 5) experiments have demonstrated that the infinite linear elastic-plastic boundary is the only static stable constitutive behavior boundary. Therefore, the yield strength and fracture strength at any constant temperature (from low temperature to high temperature) can only be defined at the static stable boundary (Fig. 7a), and we have established a general engineering allowable stress theory.

Based on classical linear elastic-plastic constitutive behavior (Fig. 6a), the academic community established a linear elastic constitutive model in the 19th century and plastic mechanics and dislocation theory in the 20th century. The classical methodology involves discovering new physical phenomena (constitutive behavior), and then modeling and simulating them. Due to insufficient understanding of the constitutive behavior of metals at constant high temperatures, structural stability issues have arisen [8]. The core issue of structural stability is the engineering allowable stress, which can only be measured from an experimental perspective. Different constant temperatures correspond to different engineering allowable stresses. This is also why Urbach et al. [8] and Cao et al. [21] were unable to address structural stability issues solely from a simulation perspective.

Taking the 45 # steel in this study as an example, at 500 °C, the creep stress ranges from 0 to 120 MPa, indicating viscoelastic creep and maintaining stability. The creep stress is between 120 and 141 MPa, which is viscoelastic and viscoplastic creep and maintains stability. Creep stress greater than 141 MPa indicates viscoelastic and viscoplastic creep instability and tendency towards fracture. At 550 °C, the creep stress ranges from 0 to 70 MPa, indicating viscoelastic creep and maintaining stability. The creep stress is between 70 and 82 MPa, which is viscoelastic and viscoplastic creep and maintains stability. Creep stress greater than 82 MPa indicates viscoelastic and viscoplastic creep instability and tendency towards fracture. Our contribution is to establish a general constitutive behavior framework for metal materials under constant temperature conditions (from room temperature to high temperature) from an experimental perspective (Fig. 7a), which solves the structural stability problem that has plagued the academic community for many years.

The traditional view is that plastic deformation occurs only when the load on metal at constant room temperature is greater than the yield stress. The view that yield depends on stress is deeply ingrained in public minds and is naturally used as a boundary for viscoelastic and viscoplastic zones [12,14]. However, our experiment showed that yield only depends on strain, although the experimental results exceeded our expectations (Figs. 2a & 3). It was demonstrated from the perspective of energy analysis that there is no constitutive behavior of "viscoplastic stress relaxation"(Appendix A). Our contribution is to demonstrate that the yield of solid materials does not depend on stress, but only on strain.

In classical linear elastic-plastic theory, elasticity and plasticity are strengthening superposition constitutive relations (Fig. 6a). However, the constitutive modeling reported in literature [12,14,33] is an independent superposition of viscoelasticity and viscoplasticity, which has certain limitations and does not fully describe the constitutive behavior of the material. Viscoelasticity and viscoplasticity are independent of each other and do not affect each other, indicating the existence of "viscoplastic stress relaxation" constitutive behavior (Appendix A), thereby assuming non conservation of energy. Therefore, the independent superposition modeling method [12,14] conflicts with the law of conservation of energy.

Our contribution is to reveal that viscoelasticity and viscoplasticity are still strengthening superposition constitutive relations, which is a new modeling direction. Prior to this study, there was no theoretical description and model characterization of the stress relaxation constitutive behavior occurring in the viscoplastic zone. Our contribution is to clearly define the constitutive behavior of "linear viscoelastic strengthening stress relaxation" (Figs. 7b & 11a) and establish a general stress relaxation constitutive behavior based on Maxwell's work. And combining Maxwell's linear viscoelastic theory and classical linear elastic-plastic theory, a general Maxwell visco-elasto-plastic fluid constitutive model was proposed to characterize linear viscoelastic strengthening stress relaxation (Fig. 8a).

The studies by Wu et al. [12] and Khayatzadeh et al. [14] suggest that viscoplastic constitutive behavior occurs when the creep constant stress exceeds the yield stress. However, this is different from our experimental results. The discovery of yield strain as the boundary between linear viscoelasticity and viscoplasticity has changed the essence of the superposition of viscoelasticity and viscoplasticity. We have determined that viscoplastic constitutive behavior only occurs when the creep strain is greater than the yield strain (ε_s). To the best of our knowledge, our experimental data is one of the few reported experimental results with inflection points in the creep curve (Fig. 12b). Our contribution is to clearly define the constitutive behavior of "linear viscoelastic strengthening creep" (Fig. 7c and d) and establish a general creep constitutive behavior based on Kelvin's work. The general creep constitutive behavior (Fig. 7) provides a new evaluation dimension for the development of new creep-resistant alloys [2,4,34]. We further confirmed that in the phenomenological construction model, the spring and friction component are inseparable (Fig. 6a). For this purpose, we propose the Kelvin visco-elasto-plastic solid constitutive model (Fig. 8b), which can be used to accurately reflect the strengthening superposition constitutive relation.

The general constitutive behavior theory constructed in this study comprehensively integrates the classical theories of Hooke, Cauchy, Maxwell, Kelvin, Voigt. Relatively insufficient is that we only derived the one-dimensional constitutive equation from the physical configuration, without deriving the three-dimensional tensor constitutive equation. This is a key point that needs further research in the future. The generalized Kelvin visco-elasto-plastic solid model has good predictive ability in solid mechanics. When the creep constant stress is greater than the fracture strength, highly nonlinear creep fluid behavior usually occurs, ultimately leading to material fracture. This highly nonlinear fluid behavior cannot be fully characterized by Newtonian fluid dampers (η_∞). This is also the reason why we only fitted a set of

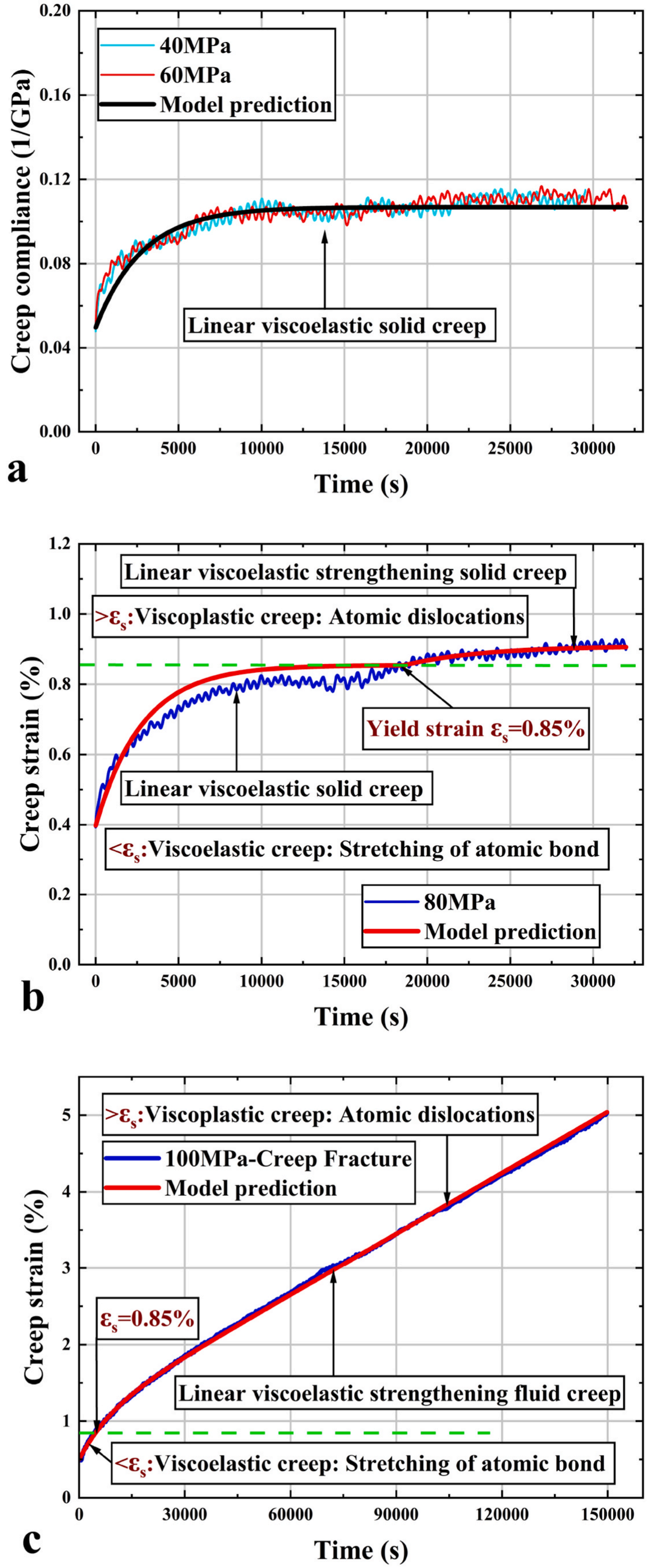

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Fig. 12. Creep constitutive behavior and model prediction. (**a**). The comparison between the fitting cure of generalized Kelvin viscoelastic solid model and the experimental data of creep compliance. (**b**). The comparison between the fitting cure of generalized Kelvin visco-elasto-plastic solid model and the experimental data of (linear viscoelastic creep (ε_s)→ linear viscoelastic strengthening solid creep) constitutive behavior. (**c**). The comparison between the fitting cure of generalized Kelvin visco-elasto-plastic fluid model and the experimental data of (linear viscoelastic creep (ε_s)→ linear viscoelastic strengthening fluid creep) constitutive behavior.

Table 4
Generalized Kelvin visco-elasto-plastic model parameters (ε_s = 0.85%).

E_0 (MPa)	δ_0			E_1^c (MPa)
	K_0 (MPa)	l_0 (MPa)	m_0 (100 %)	
20120.7	364.326	17307.03	4.11422E-32	17519.3
η_1^c (MPa •s)	δ_1^c			η_∞ (MPa •s)
	K_1 (MPa)	l_1 (MPa)	m_1 (100 %)	
49179634	331.735	257.360	3.3697E-18	376265191

creep unstable fluid constitutive behaviors (Fig. 12c). There are literature reports on the characterization of constitutive behavior of multiple creep unstable fluids using Abel dashpot [12]. This study mainly focuses on predicting the delayed stability of structures (creep solid stable constitutive behavior).

High temperature structural materials have become key materials in the fields of national defense and civil industry [35]. Establishing a theoretical framework for predicting the creep stability and fracture failure of key components at any high temperature has important engineering application significance. The design life of nuclear reactors [36] generally ranges from a few years to several decades, for example, fast neutron reactors are generally maintained at 550 °C [6,7]. The static creep fracture of metals is a way of nuclear leakage, which damages the natural environment and causes human radiation hazards. It is necessary to predict the structural delay stability of nuclear facilities. The metal containers in chemical plants [37] undergo chemical reactions in constant high-temperature and high-pressure environments, and the storage safety of various chemicals is becoming increasingly severe. Predicting the delayed stability of metal devices for quantitatively storing chemicals is the theoretical basis for preventing chemical explosions. The yield strength and fracture strength (engineering allowable stress) at any constant temperature proposed in this study can be directly applied to engineering problems [38] such as nuclear engineering and chemical plants.

7. Conclusions

In this study, we proposed and validated a theoretical framework for predicting structural stability of metals in constant high-temperature engineering environments through experimental mechanics methods. The main conclusions are as follows.

(1). Based on classical linear elastic-plastic constitutive behavior theory (from low temperature to room temperature), we have established a general constitutive behavior theory (from room temperature to high temperature) of metals.
(2). Any static constitutive behavior (including relaxation and creep) either converges to an infinite linear elastic-plastic static stable boundary, otherwise it does not converge but tends to fracture. Infinite linear elastic-plastic boundary is the only static stable boundary for measuring structural stability and fracture failure judgment.
(3). We have defined the yield strength and fracture strength (engineering allowable stress) at any constant temperature. We have determined that the yield of solid materials does not depend on stress, but only on strain.
(4). Based on the yield strain, yield strength, and fracture strength at a constant temperature, we have achieved the unity of stress relaxation and creep constitutive behavior.
(5). We revealed that viscoelasticity and viscoplasticity are strengthening superposition constitutive relations rather than independent superposition, thus establishing Maxwell fluid and Kelvin solid visco-elasto-plastic models.

The general constitutive behavior of metallic materials can be applied to various constant high-temperature engineering environments.

CRediT authorship contribution statement

Jinlai Zhou: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Bo Peng:** Writing – review & editing, Supervision. **Qinghe Yao:** Writing – review & editing, Supervision. **Gengchao Yang:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare no competing financial interests.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.msea.2025.148163.

Data availability

Data will be made available on request.

References

[1] M.A. Meyers, K.K. Chawla, Mechanical Behavior of Materials, Cambridge Univ. Press, 2008.
[2] K.A. Darling, M. Rajagopalan, M.A. Komarasamy, B.C. Hornbuckle, R.S. Mishra, K. N. Solanki, Extreme creep resistance in a microstructurally stable nanocrystalline alloy, Nature 537 (7620) (2016) 378–381.
[3] R. Wang, S. Guo, H. Chen, J. Wen, X. Zhang, S. Tu, Multi-axial creep-fatigue life prediction considering history-dependent damage evolution: a new numerical procedure and experimental validation, J. Mech. Phys. Solid. 131 (2019) 313–336.
[4] B. Zhang, Y. Tang, Q. Mei, X. Li, K. Lu, Inhibiting creep in nanograined alloys with stable grain boundary networks, Science. 378 (6620) (2022) 659–663.
[5] P. Wang, L. Cui, M. Lyschik, A. Scholz, C. Berger, M. Oechsner, A local extrapolation based calculation reduction method for the application of constitutive material models for creep fatigue assessment, Int. J. Fatig. 44 (2012) 253–259.
[6] G. Hayner, R. Bratton, R. Wright, W. Windes, T. Totemeier, K. Moore, et al., Next Generation Nuclear Plant Materials Research and Development Program Plan, Idaho National Laboratory (INL), Idaho Falls, 2005. Report No 39009143.

[7] S. Guo, J. Gong, P. Zhao, F. Xuan, A probabilistic framework of creep life assessment of structural components at elevated temperatures, Eng. Fract. Mech. 281 (5) (2023) 109162.

[8] E. Urbach, E. Efrati, Predicting delayed instabilities in viscoelastic solids, Sci. Adv. 6 (36) (2020).

[9] K. Naumenko, E. Gariboldi, Experimental analysis and constitutive modeling of anisotropic creep damage in a wrought age-hardenable Al alloy, Eng. Fract. Mech. 259 (2022) 108119.

[10] G. Kang, Q. Gao, X. Yang, A visco–plastic constitutive model incorporated with cyclic hardening for uniaxial/multiaxial ratcheting of SS304 stainless steel at room temperature, Mech. Mater. 34 (2002) 521–531.

[11] C. Chen, E. Bouchbinder, A. Karma, Instability in dynamic fracture and the failure of the classical theory of cracks, Nat. Phys. 13 (12) (2017) 1186.

[12] F. Wu, H. Zhang, Q. Zou, C. Li, J. Chen, R. Gao, Viscoelastic-plastic damage creep model for salt rock based on fractional derivative theory, Mech. Mater. 150 (2020) 103600.

[13] F. Meng, M. Saed, E. Terentjev, Rheology of vitrimers, Nat. Commun. 13 (1) (2022).

[14] S. Khayatzadeh, D. Tanner c, C. Truman, P. Flewitt, D. Smith, Creep deformation and stress relaxation of a martensitic P92 steel at 650 °C, Eng. Fract. Mech. 175 (4) (2017) 57–71.

[15] C. Su, E. Herbert, S. Sohn, J. LaManna, W. Oliver, G. Pharr, Measurement of power-law creep parameters by instrumented indentation methods, J. Mech. Phys. Solid. 61 (2) (2013) 517–536.

[16] W. Kong, S. Ma, Y. Dai, Y. Liu, Plane stress sharp V-notch tip field in power-law creeping solids, Eng. Fract. Mech. 273 (2022) 108755.

[17] P. Goel, P. Kumar, V. Jayaram, Estimation of stress redistribution for the extraction of power law creep parameters from bending, Mech. Mater. 177 (2023) 104518.

[18] P. Cui, W. Guo, A predicting model for three-dimensional crack growth in power-law creeping solids, J. Mech. Phys. Solid. 168 (3) (2022) 105029.

[19] Y.E. Beygelzimer, A.V. Spuskanyuk, The thick yield surface: an idea and approach for investigating its structure, Philos. Mag. A 79 (10) (1999) 2437–2459.

[20] R. Hill, Mathematical Theory of Plasticity, Oxford University Press, 1950.

[21] P. Cao, M. Short, S. Yip, Understanding the mechanisms of amorphous creep through molecular simulation, P. Natl. Acad. Sci. USA. 114 (52) (2017) 201708618.

[22] M. Santer, Self-actuated snap back of viscoelastic pulsing structures, Int. J. Solid Struct. 47 (2010) 3263–3271.

[23] A. Brinkmeyer, M. Santer, A. Pirrera, P.M. Weaver, Pseudo-bistable self-actuated domes for morphing applications, Int. J. Solid Struct. 49 (2012) 1077–1087.

[24] N.J. Hoff, Creep buckling, Aeronaut. Q. 7 (1956) 1–20.

[25] B. Hayman, Aspects of creep buckling. I. The influence of post-buckling characteristics, Proc. R. Soc. Lond. A Math. Phys. Eng. Sci. 364 (1978) 393–414.

[26] J. Zhou, Y. Tan, Y. Song, X. Shi, X. Lian, C. Zhang, Viscoelastic mechanical behavior of periodontal ligament: creep and relaxation hyper-viscoelastic constitutive models, Mech. Mater. 163 (2021) 104079.

[27] M. Chen, Elasticity and Plasticity, Science Press, 2007.

[28] J. Zhou, Y. Song, X. Shi, C. Zhang, Tensile creep mechanical behavior of periodontal ligament: a hyper-viscoelastic constitutive model, Comput. Methods Progr. Biomed. 207 (2021) 106224.

[29] I.Z. Oskui, A. Hashemi, Dynamic tensile properties of bovine periodontal ligament: a nonlinear viscoelastic model, J. Biomech. 49 (5) (2016) 756–764.

[30] N. Traiforos, T. Turner, P. Runeberg, D. Fernass, D. Chronopoulos, F. Glock, G. Schuhmacher, D. Hartung, A simulation framework for predicting process-induced distortions for precise manufacturing of aerospace thermoset composites, Compos. Struct. 275 (14) (2021) 114465.

[31] D. Bischoff, T. Lee, K. Kang, J. Molineux Jr., J. Pyun, M. Mackay, Unraveling the rheology of inverse vulcanized polymers, Nat. Commun. 14 (1) (2023).

[32] Z. Xu, J. Qiao, J. Wang, E. Pineda, D. Crespo, Comprehensive insights into the thermal and mechanical effects ofmetallic glasses via creep, J. Mater. Sci. Technol. 99 (2022) 39–47.

[33] W. Zhou, L. Liu, X. Lan, J. Leng, Y. Liu, Thermomechanical constitutive models of shape memory polymers and their composites, Appl. Mech. Rev. 75 (2) (2023).

[34] T. Wu, D. Dunand, Effect of eutectic microstructure on load transfer and creep resistance in Al-Ce alloys, Mat. Sci. Eng. A-Struct. 916 (2024).

[35] Y. Wan, Y. Cheng, Y. Chen, Z. Zhang, Y. Liu, H. Gong, B. Shen, X. Liang, A nitride-reinforced nbmotawhfn refractory high-entropy alloy with potential ultra-high-temperature engineering applications, Engineering 30 (2023) 110–120.

[36] N. Touran, J. Gilleland, G. Malmgren, C. Whitmer, W. Gates III, Computational tools for the integrated design of advanced nuclear reactors, Engineering 3 (2017) 518–526.

[37] Q. Cui, R. Zhao, T. Wang, S. Zhang, Y. Huang, Y. Gu, D. Xu, A 150 000 t•aˆ(-1) post-combustion carbon capture and storage demonstration Project for coal-fired power plants, Engineering 14 (2022) 22–26.

[38] G. Wu, A trio of commercial aircraft developments in China, Engineering 7 (2021) 424–426.